

## Accurate fluxes across a salt–sugar finger interface deduced from direct density measurements

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The buoyancy flux ratio,  $r = \alpha F_T / \beta F_S$ , across a salt–sugar fingering interface is determined from accurate measurements of the density variations in a two-layer run-down experiment. Since  $r$  is close to 1, the net buoyancy flux,  $\beta F_S - \alpha F_T$ , which is proportional to  $1 - r$ , gives  $r$  with a resolution that is much improved over that of previous measurements. The value of  $r$  is found to decrease from  $r \approx 0.94 \pm 0.01$  at  $R_\rho = 1.02$  to  $r = 0.88 \pm 0.01$  at  $R_\rho = 2$ , where  $R_\rho$  is the density anomaly ratio. The latter is smaller than the previous (constant) value of 0.92. The buoyancy flux due to sugar is found to decrease rapidly with increasing  $R_\rho$ , varying approximately as  $R_\rho^{-6}$ .

### 1. Introduction

Salt fingers, which occur when warm salty water overlies cooler fresher water, are thought to drive efficient transport of salt and heat in many regions of the ocean (Turner 1974). Turner (1967) and Linden (1973) measured the properties of the heat–solute finger interface in the laboratory. Stern & Turner (1969) suggested that the processes involved can be studied more conveniently by using two solutes, such as salt and sugar, which affect the density of the solution and diffuse at different rates. The two-solute system eliminates side-wall heat losses, although the molecular diffusivities are more alike than in the heat–salt system.

When a layer of sugar solution ( $S$ , the slower diffusing component) is placed above a layer of salt ( $T$ ) solution, salt fingers develop at the interface and drive convection in each layer. The fingers release the potential energy inherent in the sugar distribution, causing a downward mass flux,  $F_S$  ( $\text{g cm}^{-2} \text{s}^{-1}$ ) of sugar and an upward mass flux  $F_T$  of salt. Turner (1965) used dimensional arguments to suggest that the flux across a double-diffusive interface takes the form (when the layer depth greatly exceeds the interface thickness)

$$\beta F_S = C(\beta \Delta S)^\ddagger, \quad (1.1)$$

where  $C = C(\alpha \Delta T / \beta \Delta S, \tau, \nu / \kappa_T)$ ,  $\tau = \kappa_S / \kappa_T$  is the ratio of molecular diffusivities,  $\nu$  is the kinematic viscosity,  $\Delta S$  and  $\Delta T$  are the concentration differences between layers,  $\alpha = \rho^{-1}(\partial \rho / \partial T)$ ,  $\beta = \rho^{-1}(\partial \rho / \partial S)$  and  $S$  and  $T$  are the concentrations in grams of solute per gram of solution. By the same dimensional argument, the mass flux ratio

$$r = \alpha F_T / \beta F_S \quad (1.2)$$

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will depend upon only  $\alpha\Delta T/\beta\Delta S$ ,  $\tau$ , and  $\nu/\kappa_T$ . Turner (1967), Linden (1973) and Schmitt (1979*a*) verified these relations for the heat-solute finger interface ( $\tau \sim 10^{-3}$ ) by assuming that the two-layer systems ran down through a series of quasi-steady states in which their interface is in equilibrium with the properties of the convecting layers.

For the sugar-salt finger interface ( $\tau \sim \frac{1}{3}$ ), Stern & Turner (1969) used density measurements and colorimetry to measure the fluxes. Lambert & Demenkow (1972) used optical rotation and refractive index in an attempt to improve the measurements. Both found a flux ratio of  $r = 0.92 \pm 0.02$  over the range  $1.1 < (\alpha\Delta T/\beta\Delta S) < 2.2$ . However, a physically important quantity is the net buoyancy flux  $\beta F_S(1-r)$ , and  $1-r$  remains uncertain by 25%. Lambert & Demenkow (1972) measured  $r$  by comparing the rates of change of  $\Delta S$  and  $\Delta T$ . However, because  $r$  is close to 1, greatly improved accuracy can be obtained by directly measuring the density change within a layer and deducing  $1-r$  from this.

Little is known about the magnitude of the sugar flux through the sugar-salt interface. Stern & Turner estimated that  $C \sim 10^{-2} \text{ g cm}^{-2} \text{ s}^{-1}$  while Lambert & Demenkow found values of  $C$  ranging from  $0.5 \times 10^{-3}$  to  $0.75 \times 10^{-3} \text{ g cm}^{-2} \text{ s}^{-1}$ , more than an order of magnitude smaller. Neither author concluded that there was any significant variation of  $C$  with  $\alpha\Delta T/\beta\Delta S$ . Lambert & Demenkow assumed that  $C$  varied only slowly with time in order to integrate the flux law (1.1) and predict the time-dependence of a rundown experiment. For the heat-solute interface, on the other hand, it is known that  $C \propto (\alpha\Delta T/\beta\Delta S)^{-\frac{1}{2}}$ .

In this paper we describe the results of experiments in which a layer of sugar solution was floated on a layer of salt solution and the system allowed to run down. Samples were withdrawn from each layer and analysed for density and solute concentrations. The experiments and analysis are described in § 2 and the results discussed in § 3. Complications introduced by the nonlinear equations of state ( $\rho\alpha$  and  $\rho\beta$  dependent upon  $S$  and  $T$ ) for our laboratory system make it more practical to replace  $\alpha\Delta T/\beta\Delta S$  with a new density ratio  $R_\rho$  (defined by 3.1). However, the two ratios are shown to be identical to within the experimental uncertainties of previous work. The flux ratio  $r$  is found not to be a constant, but to increase significantly at values of  $R_\rho$  near one. For  $R_\rho > 1.2$ ,  $r$  was found to be less than 0.90. The sugar flux coefficient,  $C$ , was found to vary by two orders of magnitude, approximately proportional to  $R_\rho^{-6}$ .

## 2. Experiments

Three experiments were carried out in a Perspex tank of horizontal cross-section  $38 \times 38$  cm. The depth of each layer,  $H = 25$  to 30 cm, was chosen to be smaller than the horizontal dimensions. This layer depth resulted in a total run-down time of about three days. A high degree of time resolution was therefore possible, even in the early parts of an experiment when rapid variations of  $S$  and  $T$  occurred.

Solutions of sodium chloride and sugar were mixed to the desired densities and left for 24 hours to come to room temperature ( $25^\circ\text{C} \pm 2^\circ\text{C}$  for all experiments). Initial densities were 10.7% (experiment 1), 5.8% (experiment 2), and 10.5% (experiment 3) greater than the density of fresh water. The specific gravity of the salt solution was greater by 0.05–0.1%. After placing the lower (salt) layer in the box, the upper layer was carefully floated on top through a diffuser. This step took 5–10 min. In all but experiment 1, the top of the tank was then sealed to reduce evaporation from the top

layer. Sampling began immediately after filling was completed, with 20 ml samples being withdrawn simultaneously from the centre of each layer at intervals which ranged from 30 s at the beginning to 24 hours in the later stages. These intervals were chosen to span approximately constant changes in layer densities and concentrations. The interface position and thickness were monitored throughout the experiment.

Despite the vigorous convection driven by the interfacial buoyancy flux, each layer had a tendency to stratify (in the salt-finger sense) near the boundaries opposite the interface. This tendency is due to the time-dependent interfacial flux. The plumes falling into the lower layer from the interface have a lower density (due to the smaller interfacial flux) than the plumes at an earlier stage of the run-down. The earlier plumes may have reached the bottom of the layer but the later plumes are unable to do so. In order to remove this stratification, very mild stirring over very short periods was accomplished, by means of a magnetic stirrer in the bottom layer and manual stirring in the top layer. Care was taken to see that the interface was not disturbed.

The density  $\rho$ , conductivity  $\lambda$ , and optical rotation  $\phi$ , of each sample was measured. A precision densimeter (Anton Paar, K.G., Austria, Model DMA 02C) gave a precision of  $4 \times 10^{-5} \text{ g cm}^{-3}$  in density at a temperature of  $25 \pm 0.01^\circ \text{C}$ . The rotation of polarized light as it passed through a 20 cm light path was measured to  $\pm 0.1^\circ$  by a Polax Polarimeter (Atago Optical Co., Tokyo). Conductivity was measured with a precision of 0.2% with a Phillips Model PW 9501/01 conductivity bridge and a two-electrode platinized cell, which was immersed in a  $20 \pm 0.1^\circ \text{C}$  bath.

From the three measurements on each sample, three different estimates of ( $S$ ,  $T$ ) could be obtained using the polynomials and inversion method of Ruddick & Shirtcliffe (1979). The estimates agreed within the expected uncertainties. However, for the upper layer, the high  $S$  and low  $T$  imply that the density and conductivity measurements yield the most accurate concentration estimates. Conversely, the density and rotation measurements were used for the lower layer. For both layers the concentrations were measured to an absolute accuracy of 0.002 for  $S$  and 0.001 for  $T$ , and to a precision one order of magnitude smaller.

### 3. Results

At this stage we note that the parameter  $\alpha\Delta T/\beta\Delta S$  is not the most suitable for systems in which  $\alpha$  and  $\beta$  are functions of  $S$  and  $T$ . When the layer densities are equal, this parameter may not be equal to 1. However, the behaviour of the interface changes rapidly when the layer densities are nearly equal, and it is the condition of equal densities that presents an absolute limit to this variation. In order to reflect this behaviour it is necessary to use a density-anomaly ratio that always approaches 1 when  $\Delta\rho \rightarrow 0$ . With our direct density measurements a practical definition for the density-anomaly ratio  $R_\rho$  is

$$R_\rho \equiv 1 + \Delta\rho/\overline{\rho\beta}\Delta S, \quad (3.1)$$

where  $\overline{\rho\beta} = \frac{1}{2}[\rho\beta(S_1, T_1) + \rho\beta(S_2, T_2)]$  is the mean  $\partial\rho/\partial S$  of the two layers.† For a linearized equation of state ( $\rho\alpha$  and  $\rho\beta$  constant),  $R_\rho$  as defined by (3.1) reduces to  $\alpha\Delta T/\beta\Delta S$ .

† An alternative is to choose  $\overline{\rho\beta} = \rho\beta(\bar{S}, \bar{T})$ , where  $\bar{S}$ ,  $\bar{T}$  are the mean concentrations. The difference is small. To replace  $\overline{\rho\beta}\Delta S$  in (3.1) with  $\rho\beta(S_1, T_1)S_1 - \rho\beta(S_2, T_2)S_2$  is not suitable as this can contain a large contribution from  $\Delta T$ .

*Flux ratio*

We can discuss the fluxes within individual layers. From conservation of salt and sugar in each layer, we have the exact relations

$$\beta F_S = \mp \rho \beta H \frac{dS}{dt}, \quad (3.2)$$

$$\alpha F_T = \pm \rho \alpha H \frac{dT}{dt}, \quad (3.3)$$

where the upper (lower) sign refers to the upper (lower) layer, the layer depths are assumed to be constant and  $\rho$ ,  $\alpha$ ,  $\beta$  take the local values.  $F_S$  and  $F_T$  are both positive mass fluxes. The buoyancy flux ratio  $r = \alpha F_T / \beta F_S$  for an individual layer can be found from

$$r = -\frac{\alpha}{\beta} \frac{dT}{dS}. \quad (3.4)$$

However,  $dT/dS$  cannot be estimated to better than a few per cent, giving a similar uncertainty in  $r$ . In addition,  $\alpha/\beta$  is different for each layer and varies during an experiment (by 1% in our experiments).

Direct measurements of density allow us to use a slightly different method. The net density change in each layer is

$$\begin{aligned} \frac{d\rho}{dt} &= \frac{\partial \rho}{\partial S} \frac{\partial S}{\partial t} + \frac{\partial \rho}{\partial T} \frac{\partial T}{\partial t} \\ &= \rho \beta \frac{\partial S}{\partial t} + \rho \alpha \frac{\partial T}{\partial t} \\ &= \rho \beta \frac{\partial S}{\partial t} (1 - r), \end{aligned} \quad (3.5)$$

where (3.2)–(3.4) have been used in the last step. Therefore

$$1 - r = \frac{1}{\rho \beta} \frac{d\rho}{dS}. \quad (3.6)$$

This is true for each layer separately;  $r$  found by this method could conceivably be different for each layer. Because  $r$  is close to 1, even a 10% error in the right-hand side of (3.6) gives only a 1% error in  $r$ . Variations in  $r$  can be easily detected with this method.

In figure 1 are plotted the  $\rho$  versus  $S$  data for the three experiments, upper layers ( $U$ ) in the upper half, lower layers ( $L$ ) in the lower half. (The curves have been shifted an arbitrary amount on both axes.) The small arrows show the direction of increasing time. The broken line indicates the slope corresponding to  $r = 0.91$  (the result of a straight-line fit by eye to the  $3L$  curve). However, the increase in slope (i.e. a decrease in  $r$ ) as time progresses is obvious. The effects of evaporation from the upper layer of experiment 1 are apparent, since the density passes through a minimum. No influence on the bottom layer could be seen.

The slopes  $\partial\rho/\partial S$  of the data in figure 1 were estimated in a smoothed fashion by least-squares fitting each set of data to a curve of the form  $\rho = a + bS + cS^2$ , and then evaluating the derivative of the smoothed curve at the data points. The curves which

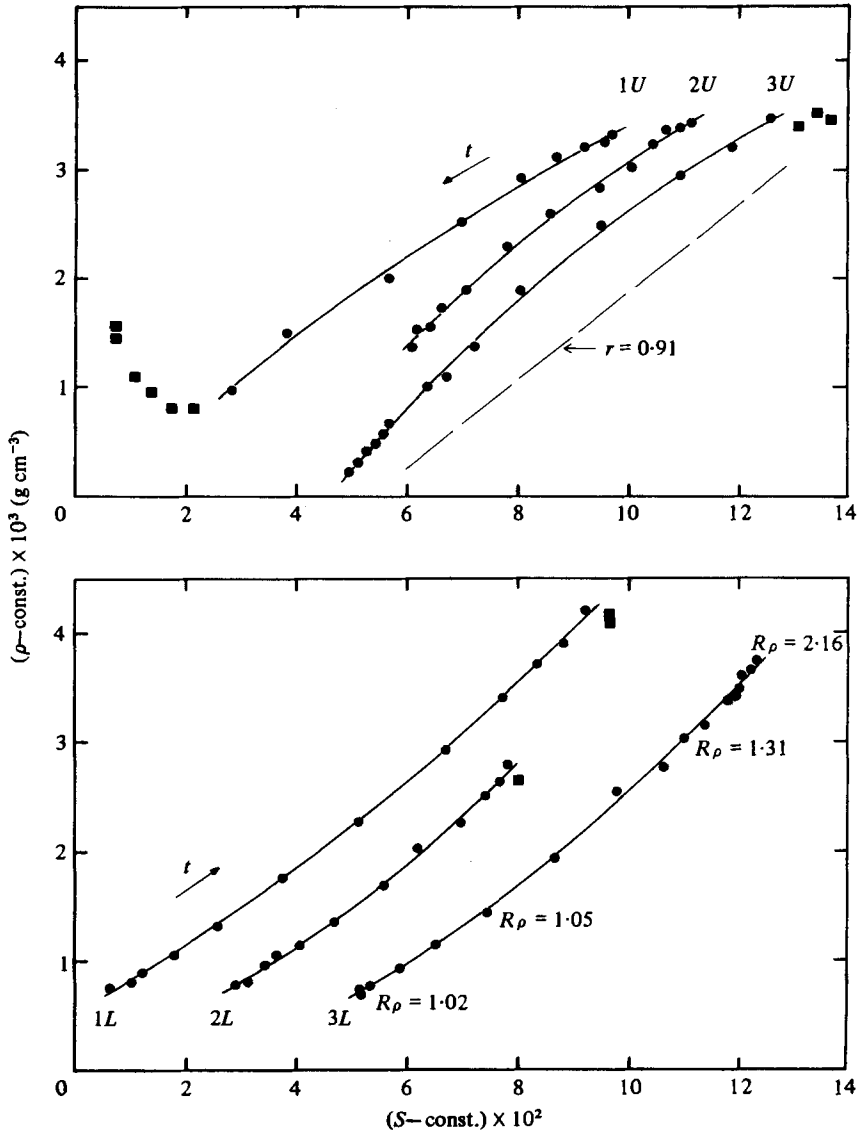


FIGURE 1. Measured density of the samples, plotted against computed sugar concentration. Each curve has been shifted on each axis by an arbitrary amount. Individual experiments are numbered, *U* refers to the upper layer and *L* to the lower layers. The directions of increasing time (*t*) are shown. The curved lines are quadratic fits to the data; points shown as squares were rejected. The broken line corresponds to a flux ratio of  $r = 0.91$ . Values of the density-anomaly ratio are marked for one layer of experiment 3.

were actually fitted to the data are shown as solid lines in figure 1. The method works well only if the smoothed curve fits the points globally; the form chosen seems to be suitable. However, it was found that a few points with unusually large deviations (the squares in figure 1) could affect the fit strongly over the whole range of data. Those points were not used in the fitting procedure.

The slopes from the smoothed curves were used to evaluate  $r$  at the values of  $S$

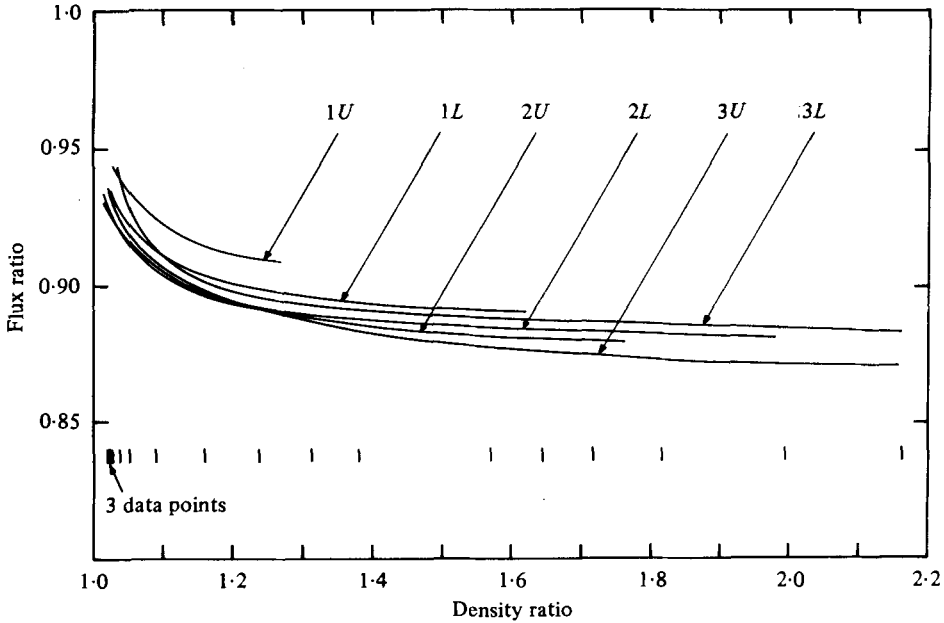


FIGURE 2. The buoyancy flux ratio,  $r$  [equation (1.2)], as a function of the density-anomaly ratio,  $R_\rho$  [equation (3.1)], for individual layers. Layers are marked as in figure 1. The density of data points is shown by the vertical marks.

corresponding to the data points. The values of  $R_\rho$  and  $\bar{\alpha}\Delta T/\bar{\beta}\Delta S$  at these  $S$  values were also calculated from the measurements. The two density ratios were found to differ by mean values of 0.3% at  $R_\rho \sim 1$  and 2% near  $R_\rho = 2$ . In figure 2 is plotted the flux ratio  $r$  versus the density-anomaly ratio  $R_\rho$  for both layers of the three experiments. The curve from the upper layer of experiment 1 (1U), differs markedly from the others, presumably owing to the effects of evaporation. The other curves all agree well, with a scatter in  $r$  of  $\pm 0.01$ , and no systematic difference between the upper and lower layers of an experiment. The flux ratio varies systematically with  $R_\rho$ , starting with high values  $r = 0.94 \pm 0.01$  at  $R_\rho = 1.02$  and decreasing to a nearly constant  $r = 0.88 \pm 0.01$  for  $R_\rho > 1.6$ . Note that, for large  $R_\rho$ , the value of  $1-r$  ( $0.12 \pm 0.01$ ) is 50% larger than previous estimates of  $0.08 \pm 0.02$ . As will be seen in the next paragraph, the value obtained by previous authors is a result of the method used to obtain the flux ratio.

Lambert & Demenkow (1972) measured the refractive index and optical rotation to determine  $S$  and  $T$  in each layer. From the conservation equations (3.2) and (3.3), the flux ratio can be written in the form

$$r = \frac{\bar{\alpha}(\rho_1 H_1 \partial T_1 / \partial t - \rho_2 H_2 \partial T_2 / \partial t)}{\bar{\beta}(\rho_2 H_2 \partial S_2 / \partial t - \rho_1 H_1 \partial S_1 / \partial t)} \approx \frac{\bar{\alpha} d(\Delta T)}{\bar{\beta} d(\Delta S)}, \quad (3.7)$$

where  $\bar{\alpha}$  and  $\bar{\beta}$  are the mean  $\alpha$  and  $\beta$  of the two layers and subscripts 1 and 2 refer to the upper and lower layers. Note that equation (3.7) is only accurate to the extent that

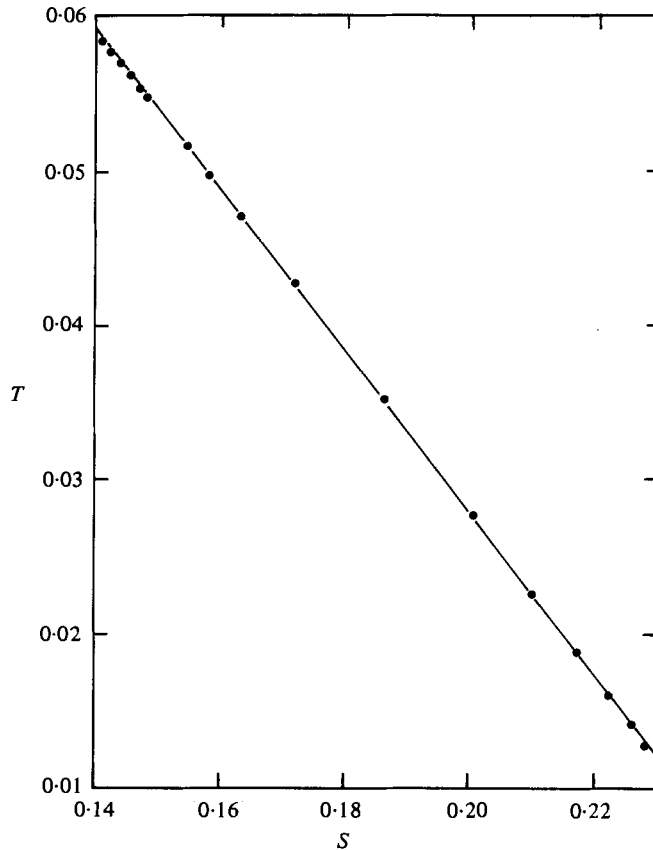


FIGURE 3. The concentration of salt ( $T$ ) as a function of the sugar concentration ( $S$ ) in grams of solute per gram of solution for the lower layer of experiment 3. The straight line of best fit corresponds to a flux ratio of  $r = 0.915 \pm 0.01$ .

$\rho_1 H_1 \approx \rho_2 H_2$ . Lambert & Demenkow plotted  $\Delta T$  versus  $\Delta S$  and obtained the value  $r = 0.92 \pm 0.02$  from the mean slope. (Their data were too noisy to show that  $r$  was anything but constant.) In a similar approach, we have plotted, in figure 3,  $T$  versus  $S$  from the lower layer of experiment 3. From equation (3.4), without assuming equal layer depths or densities, the slope of the graph is proportional to  $r$ . Although a slight curvature is visible, the data almost appear to be consistent with a straight line, and a fit by eye to the points gives  $r = 0.915$ . Note that this 'average' value was computed using the same data from figure 1 which clearly shows a change in  $r$  of more than 0.05 from beginning to end of the experiment. Thus the straight-line fit to the  $T, S$  plot has resulted in a mean flux ratio estimate that is weighted towards the lower values of  $R_\rho$ , where  $r$  is larger. This suggests that the data of Lambert & Demenkow are consistent with our flux ratio results, since analysing our data as in figure 3 gives results similar to those of Lambert & Demenkow. Stern & Turner (1969) used equation (3.6) as we did, but their experimental error was too large to allow them to resolve changes in  $r$ .

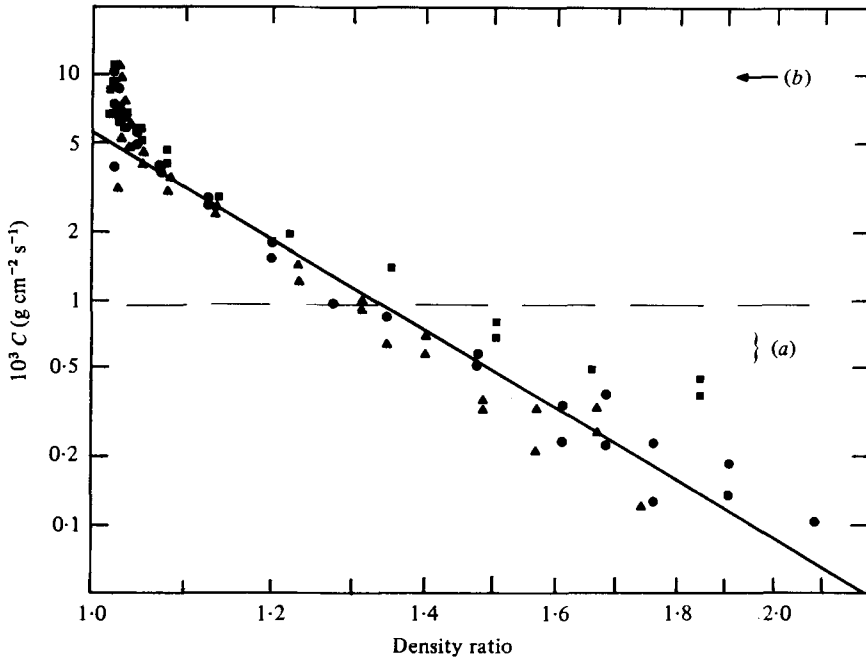


FIGURE 4. The buoyancy flux coefficients for sugar,  $C = \bar{\beta} F_S (\bar{\beta} \Delta S)^{-\frac{4}{3}}$ , found from (3.8) for the salt-sugar finger interface.  $\blacktriangle$ , experiment 1,  $\beta \Delta S = 10.7\%$ ;  $\blacksquare$ , experiment 2,  $\beta \Delta S = 5.8\%$ ;  $\bullet$ , experiment 3,  $\beta \Delta S = 10.5\%$ . Results for both upper and lower layers are shown. The solid line is a fit by eye:  $C = 5.5 \times 10^{-3} R_\rho^{-6}$ . The broken line is the 'solid plane' value,  $C^{SP}$ , given by (3.10). Estimates of  $C$  obtained by (a) Lambert & Demenkow (1972), and (b) Stern & Turner (1969) are also shown.

#### Sugar flux

Estimates of the sugar concentrations in each layer yielded values for the density (or buoyancy) flux due to sugar. Using (3.2) and (1.1) we define the coefficient

$$C = \overline{\rho\beta} H (\delta S / \delta t) (\bar{\beta} \Delta S)^{-\frac{4}{3}}, \quad (3.8)$$

where  $\overline{\rho\beta}$  and  $\bar{\beta}$  are average values for the two layers. The quantity  $\delta S$  is the change in  $S$  between samples that were withdrawn at time intervals  $\delta t$  apart and  $H$  is the layer depth during each interval.

Values of  $C$  were found for each layer of each experiment. These are plotted as a function of the density anomaly ratio,  $R_\rho$ , in figure 4. No systematic differences between upper and lower layers were detected. It was assumed that the  $\frac{4}{3}$ -power law of (1.1) is valid, a dependence that was shown to be true for the heat-salt finger interface (Schmitt 1979a). While the systematic difference between the values of  $C$  for experiment 2 ( $\beta \Delta S \approx 5\%$ ) and those for  $\beta \Delta S \approx 10\%$  are only of the same magnitude as the scatter for individual experiments, more experiments are needed to test the assumed dependence [equation (1.1)] upon  $\Delta S$ . The straight line in figure 4 shows the power law

$$C = 5.5 \times 10^{-3} (\text{g cm}^{-2} \text{s}^{-1}) R_\rho^{-6}, \quad (3.9)$$

which provides a reasonable fit to the data for  $R_\rho \geq 1.05$ . At lower values of the density anomaly ratio the sugar flux varies much more rapidly with  $R_\rho$ .

The flux coefficient,  $C$ , varies by over two orders of magnitude. The (constant) values estimated by Lambert & Demenkow and Stern & Turner are shown on figure 4



as (a) and (b) respectively. The data are also compared to the value of the flux coefficient,  $C^{SP}$ , that describes the transport of sugar by high-Rayleigh-number convection from a solid but perfectly permeable, horizontal membrane;

$$C^{SP} = 0.085(g/\nu)^{\frac{1}{2}} \kappa_S^{\frac{3}{2}}. \quad (3.10)$$

The sugar flux coefficient for the finger interface falls below this 'solid plane' value (broken line on figure 4) when  $R_\rho > 1.3$ . Fingers within a heat-salt interface, on the other hand, yield a value of  $C$  that is always greater than 10 times the appropriate 'solid plane' value.

#### 4. Discussion of results

It can be seen in figure 1 that the flux ratio  $r$  tends to increase (smoothly) with decreasing density ratio,  $R_\rho$ . The rate of increase of  $r$  becomes greater as  $R_\rho$  approaches 1. The reasons for this variation are not yet understood, although such an increase of the flux ratio as  $R_\rho \rightarrow 1$  has already been suggested by a model for the steady finger interface (Schmitt 1979*b*). This model also successfully predicts the observed variation of flux ratio between the heat-salt and salt-sugar systems (a difference of Prandtl number  $\nu/\kappa_T$  and diffusivity ratio  $\tau$ ). However, the model predictions depend explicitly upon the initial conditions ( $R_\rho$ ) at the time the steady interface is formed from two uniform layers. The interface in a laboratory experiment, on the other hand, passes through a series of quasi-steady (equilibrium) states as the interfacial fluxes reduce the concentration differences between convecting layers, and it is known that the properties of the interface do not depend upon initial conditions. They depend only upon the current concentration differences across the interface. The agreement between prediction and experiment may therefore be fortuitous.

Observations of the salt-sugar interface suggest another possible contribution to the increasing flux ratio at density ratios close to 1. Figure 5 (plate 1) shows two shadowgraph photographs of an interface at (a)  $R_\rho \sim 1.03$  and (b)  $R_\rho \sim 1.7$ . In figure 5(b) there can be seen vertical quasi-steady convection cells in a region that is much deeper than the width of an individual cell. Only the ends of the fingers (in the 'transition zone') are influenced by the horizontal velocities of the mixed-layer convection. On the other hand, when  $R_\rho \rightarrow 1$  (figure 5*a*) the interface thickness is too small ( $< 0.1$  cm) to be resolved by the shadowgraph. The dynamics of the short interfacial fingers might then be influenced by strong viscous communication between the turbulent convective motions in the mixed layers and the fluid within the interface. This may lead to intermittent destruction and regrowth of the fingers. In the limit, convection would maintain a density discontinuity between periods of salt-finger growth. Fingers of maximum growth rate forming from such a step are predicted to yield  $r \rightarrow 1$  when  $R_\rho \rightarrow 1$  (Schmitt 1979*b*). This behaviour of the ratio of buoyancy fluxes is consistent with that observed in our experiments. For laboratory heat-solute interfaces no distinct increase of  $r$  has been detected at low density ratios and the interface thickness has always been greater than several centimetres. The variation in flux ratio for salt-sugar systems might therefore be due to a change in the type of salt fingers (from steady to maximum growth rate) as  $R_\rho \rightarrow 1$ .

Our measurements yield a flux coefficient  $C$  for sugar that is strongly dependent upon the density anomaly ratio  $R_\rho$  and that is one to two orders of magnitude smaller

than the coefficient previously measured for heat-solute finger interfaces (see Linden 1973; Schmitt 1979*a*). The difference between the heat-solute and two-solute systems is largely due to the Prandtl number  $\nu/\kappa_T$ , which is two orders of magnitude larger in the salt-sugar system. Linden has shown that  $C = (\nu/\kappa_T)^{-\frac{1}{2}} f(r, R_\rho)$  in the limit  $\tau \ll 1$  and this Prandtl-number dependence will be valid for more general values of the diffusivity ratio. On the other hand, a prediction of the dependence of the flux coefficient and interface structure upon the density ratio  $R_\rho$  and the diffusivity ratio  $\tau$  must await an adequate model for the quasi-steady interface (since laminar, quasi-steady salt fingers appear to dominate the interface at most density ratios). This will require a description of the coupling between laminar salt fingers and turbulent convection at the transition zones (Griffiths 1979).

Another significant observation is that the flux of sugar through salt-sugar finger interfaces is less than the appropriate 'solid plane' flux when  $R_\rho > 1.3$ . This implies very small vertical convection velocities within the fingers. In order to estimate these velocities, the flux of  $S$  through a finger interface is given by  $F_S = \overline{\rho w' S'}$ , where  $w'$  and  $S'$  are the velocity and  $S$  perturbations of the fingers and the bar denotes a horizontal average. For sinusoidal perturbations this becomes  $F_S = \frac{1}{4} \hat{w} \hat{S}$ , where  $\hat{w}$  and  $\hat{S}$  are the salt-finger amplitudes. With the maximum possible value of  $\hat{S} = \frac{1}{2} \Delta S$ , the data of figure 4 give finger velocities in the range  $4 \times 10^{-4} < \hat{w} < 4 \times 10^{-2} \text{ cm}^{-1}$  for our salt-sugar interfaces. Smaller values of  $\hat{S}$  would imply larger values of  $\hat{w}$ , but again a better understanding of the interface is required before the horizontal variations between fingers can be predicted.

In all experiments the interface was observed to migrate slowly upward. The net entrainment of fluid into the lower layer was too slow, however, to have a strong effect on the flux ratio. We estimate a maximum influence of 17% on the apparent flux of sugar into the lower layer. Since the apparent salt flux into the lower layer is also increased, there is a maximum influence upon the flux ratio,  $r$ , of 0.5%. The interface migration may be due to unequal convection velocities in the two layers, an inequality that could be caused by unequal layer depths and viscosities, or to a net transport of fluid across the interface due to unequal molecular properties in up- and down-going salt fingers.

There is room for improvement of the measurements presented in this paper. More experiments are required to verify the  $\Delta S^{\frac{1}{2}}$  law for the flux. Within each experiment, samples could be withdrawn more often, and the extra data points would allow a more local estimation of the slope  $d\rho/dS$  by means of a convolution filter. Presumably, these improved techniques would better resolve the behaviour of the flux ratio. Another problem due to the nonlinearity of the equation of state  $\rho(S, T)$  may then arise. Since the mass of salt and sugar must be conserved,  $F_T/F_S$  must be the same for each layer. However,  $\alpha/\beta$  is different in each layer (by about 1%), so the value of  $r$  must also be different. This difference would presumably be discernable in more accurate experiments and would have to be taken into account.

## 5. Conclusions

Determination of net buoyancy fluxes by direct density measurements have allowed improved estimates of the ratio  $r$  [equation (1.2)] of buoyancy fluxes due to salt and sugar across a finger interface. The flux ratio was found to decrease

systematically with increasing density-anomaly ratio  $R_\rho$  [equation (3.1)], decreasing from

$$r = 0.94 \pm 0.01 \quad \text{at} \quad R_\rho = 1.02 \quad \text{to} \quad r = 0.88 \pm 0.01 \quad \text{at} \quad R_\rho = 2.$$

Over most of the range of  $R_\rho$ , we found that  $r$  was significantly less than the average value of  $0.92 \pm 0.02$  reported by Lambert & Demenkow (1972). The difference was found to be due to a systematic bias in their method of determining  $r$ .

The buoyancy flux due to sugar was computed for each sampling interval. The quantity  $C = \beta F_S (\beta \Delta S)^{-\frac{1}{2}}$  was found not to be a slowly varying function of time (or  $R_\rho$ ) as was assumed by Lambert & Demenkow (1972), but rather to decrease rapidly with increasing  $R_\rho$ . For  $R_\rho > 1.05$  the power law  $C = 5.5 \times 10^{-3} R_\rho^{-6}$  ( $\text{g cm}^{-2} \text{s}^{-1}$ ) provided a reasonable description of the data.

#### REFERENCES

- GRIFFITHS, R. W. 1979 Transport through thermohaline interfaces in a viscous fluid and a porous medium. Ph.D. thesis, The Australian National University.
- LAMBERT, R. B. & DEMENKOW, J. W. 1972 On vertical transport due to fingers in double diffusive convection. *J. Fluid Mech.* **54**, 627-640.
- LINDEN, P. F. 1973 On the structure of salt fingers. *Deep-Sea Res.* **20**, 325-340.
- RUDDICK, B. R. & SHIRTCLIFFE, T. G. L. 1979 Data for double diffusers: Physical properties of aqueous salt-sugar solutions. *Deep-Sea Res.* **26**, 775-787.
- SCHMITT, R. W. 1979a Flux measurements on salt fingers at an interface. *J. Mar. Res.* **37**, 419-436.
- SCHMITT, R. W. 1979b The growth rate of super-critical salt fingers. *Deep-Sea Res.* **26**, 23-40.
- SHIRTCLIFFE, T. G. L. 1973 Transport and profile measurements of the diffusive interface in double diffusive convection with similar diffusivities. *J. Fluid Mech.* **57**, 27-43.
- STERN, M. E. & TURNER, J. S. 1969 Salt fingers and convecting layers. *Deep-Sea Res.* **16**, 497-511.
- TURNER, J. S. 1965 The coupled turbulent transports of salt and heat across a sharp density interface. *Int. J. Heat Mass Transfer* **8**, 759-767.
- TURNER, J. S. 1967 Salt fingers across a density interface. *Deep-Sea Res.* **14**, 559-611.
- TURNER, J. S. 1974 Double-diffusive phenomena. *Ann. Rev. Fluid Mech.* **6**, 37-56.

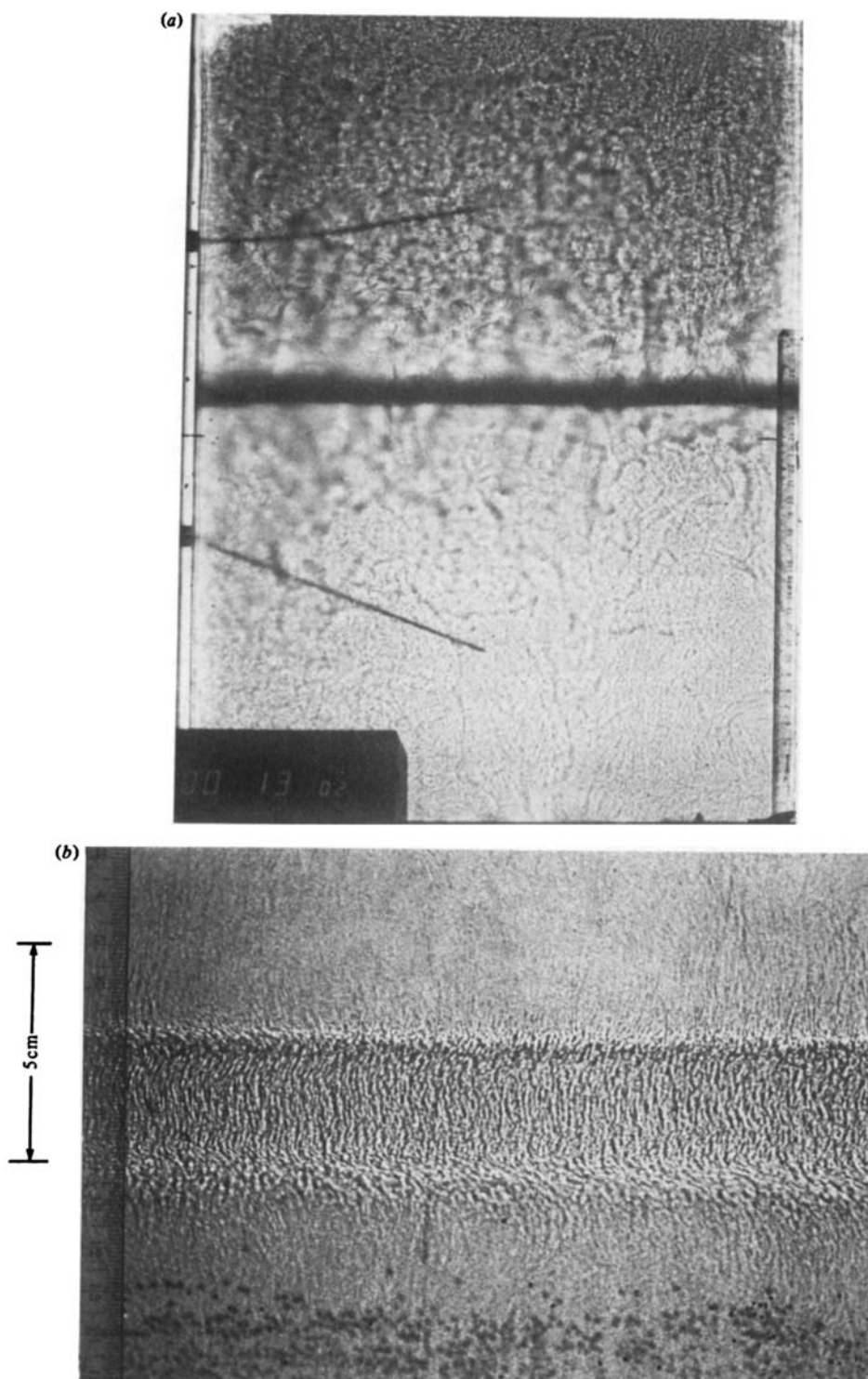


FIGURE 5. Shadowgraph photographs of the salt-sugar finger interface at (a)  $R_\rho \sim 1.03$  and (b)  $R_\rho \sim 1.7$  ( $\beta\Delta S = 5.8\%$ ).